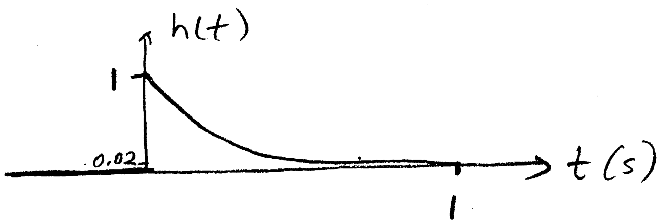
Signals & Systems
Quiz 3 Solution

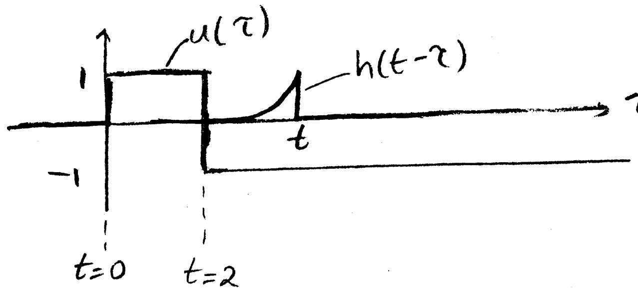
SS1 (a) Time constant $= RC = \frac{1}{4}$ sec

(b)



$h(t) = e^{-4t}\,\sigma(t)$

(c)



Region of nonzero overlap depends on t

$t<0$: $\quad y(t) = 0$

$0<t<2$: $\quad y(t) = \int_0^t e^{-4(t-\tau)}\,d\tau = \left[\frac{1}{4}e^{-4(t-\tau)}\right]_0^t = \frac{1}{4}(1-e^{-4t})$

$t>2$:

$$y(t) = \int_0^2 e^{-4(t-\tau)}\,d\tau - \int_2^t e^{-4(t-\tau)}\,d\tau$$

$$= \left[\frac{1}{4}e^{-4(t-\tau)}\right]_0^2 - \left[\frac{1}{4}e^{-4(t-\tau)}\right]_2^t$$

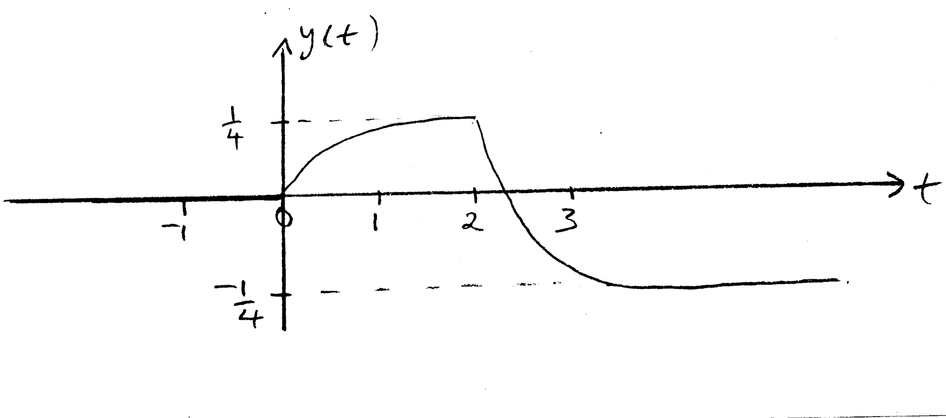
$$= \frac{1}{4}\left(e^{-4(t-2)} - e^{-4t} - 1 + e^{-4(t-2)}\right)$$

$$= \frac{-1}{4}\left(1 + e^{-4t} - 2e^{-4(t-2)}\right)$$

So

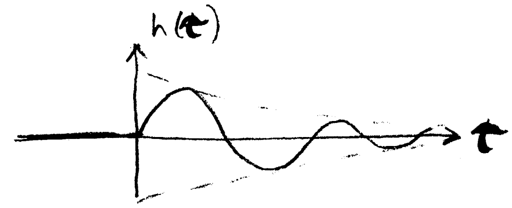
$$y(t) = \begin{cases} 0, & t<0 \\ \frac{1}{4}(1-e^{-4t}), & 0<t<2 \\ -\frac{1}{4}(1+e^{-4t}-2e^{-4(t-2)}), & t>2 \end{cases}$$

(d)



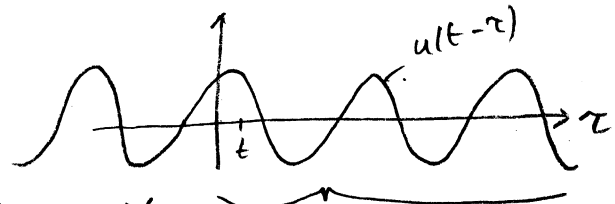
Signals & Systems
Quiz 3 solution

SS 2 (a) $\quad h(t) = e^{-\alpha t}\sin t, \; t > 0$



$u(t) = e^{j\omega t}$

$$y(t) = \int_{-\infty}^{\infty} u(t-\tau)\,h(\tau)\,d\tau$$



Euler's formula : $\quad \sin t = \dfrac{e^{jt} - e^{-jt}}{2j}$

nonzero overlap for $\tau = 0$ to $\tau = \infty$ for all t

$$y(t) = \int_0^{\infty} e^{j\omega(t-\tau)} e^{-\alpha\tau} \frac{\left(e^{j\tau} - e^{-j\tau}\right)}{2j}\, d\tau$$

$$= \frac{e^{j\omega t}}{2j} \int_0^{\infty} \left(e^{(-\alpha - j\omega + j)\tau} - e^{(-\alpha - j\omega - j)\tau} \right) d\tau$$

$$= \frac{e^{j\omega t}}{2j} \left[\frac{e^{(-\alpha - j\omega + j)\tau}}{-\alpha - j\omega + j} - \frac{e^{(-\alpha - j\omega - j)\tau}}{-\alpha - j\omega - j} \right]_0^{\infty}$$

$$= \frac{e^{j\omega t}}{2j} \left[\frac{1}{-\alpha - j(\omega + 1)} - \frac{1}{-\alpha + j(1 - \omega)} \right]$$

$$= \frac{e^{j\omega t}}{2j} \left[\frac{-\cancel{\alpha} + j - \omega\cancel{j} + \cancel{\alpha} + j\omega + j}{\alpha^2 - j\cancel{\alpha} + j\alpha\omega + j\alpha\omega + j\cancel{\alpha} + 1 - \omega^2} \right]$$

$$= \frac{e^{j\omega t}}{\alpha^2 + 2j\alpha\omega + 1 - \omega^2}$$

(b) If $\alpha = 0$, then $\omega = 1$ causes $y(t) \to \infty$

So $u(t) = e^{jt}$ is a bounded input that leads to an unbounded output
(forcing at resonant frequency for an undamped system)